

AN ALGEBRA HANDBOOK: INSTRUCTIONAL STRATEGIES FOR ASSISTING  
COLLEGE STUDENTS WITH LEARNING DISABILITIES

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PROJECT

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A Project

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Abstract  
of  
AN ALGEBRA HANDBOOK: INSTRUCTIONAL STRATEGIES FOR ASSISTING  
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An Algebra handbook of instructional strategies was created for the Learning Disabilities department staff at American River College in Northern California. The handbook is designed to be a resource tool that will allow Instructional Assistants and Learning Disability Specialists to better facilitate instructional strategies to students with learning disabilities who struggle in mathematics. The handbook provides a detailed step by step process for solving mathematical problems. Visual illustrations and tactile representations within the handbook will allow staff to present and apply basic mathematical concepts, concrete operations, relational thinking, and construct reasoning during instructional strategies sessions. The content of the handbook is the result of a comparative analysis between the current Algebra textbooks used at American River College to a teacher's edition of Algebra. This handbook enables Instructional Assistants and Learning Disability Specialists be more effective and supportive in the process of helping students and helps to promote positive academic achievement through the use of

best practices and learning strategies for students with learning disabilities at the postsecondary level.

\_\_\_\_\_, Committee Chair  
Bernice Bass de Martinez, Ph.D.

\_\_\_\_\_  
Date

## Dedication

I would like to dedicate this project to my parents Chloe and Mark Gitelson for their inspiration and guidance through all aspects of my development. I would like to thank my family and friends for their time and effort so graciously given along my journey. To my mother-in-law Elnora Crowder for her kind words. To my husband Martin and my daughter Jasmine for their unwavering support. To my brother Jerome for his unending encouragement. To my mentor Carolyn Javier for daring me to expand my vision and for helping me stay focused. Finally, to my grandmother Louise Addison who always knew I would accomplish great things.

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## CHAPTER 1

### INTRODUCTION

#### Background

Colleges and universities in Northern and Southern California are seeing a large influx of students on their campuses. As a result, the number of students with learning disabilities has also increased. “Students with learning disabilities represent the fastest growing population of college students with disabilities. They are graduating from high school and entering colleges and universities in record numbers” (Henderson, 2001, p. 1). California Community Colleges are being confronted by the magnitude of students who lack the fundamental math skills that are required to complete an Associate of Arts degree and/or advance to a four-year university (Bettinger & Long, 2009). Many students who struggle mathematically can often experience feelings of intellectual inadequacy that can erode self-esteem and academic motivation. Self-esteem is equated with self-respect and is believed to represent “a person’s sense of his own value” and “a confidence in one’s ability . . . to fulfill one’s intention” (Rawls, 1971, p. 440). Consequently, self-esteem can be considered a strong determinant and a predictor of the level of accomplishment that individuals finally attain (Christou et al., 2001, p. 45).

College and university systems have traditionally provided academic support on campus through resources such as Math Multimedia Learning Center (MMLC), Learning Resource Center (LRC), Learning Disability Department (LD) within Disabled Students Program Services (DSPS), and BEACON which provide tutoring and strategies to assist students who encounter challenges in math. Prior to 2009, graduation standards at

American River College in Sacramento, CA, permitted students to take AT 105, Math for Auto Technicians, (the equivalent to Math 100, Elementary Algebra) to fulfill the math graduation requirement. Within 112 campuses comprised of 72 districts, many of the California Community College Districts have articulation agreements with four-year universities that must be satisfied with the appropriate course requirements. The community colleges work in partnership with the University of California, the California State University, and private universities to establish a course agreement for transferring and graduating students to receive credit for their academic programs <http://web.arc.losrios.edu/catalog/MathandStats.pdf>. Currently under the new guidelines mandated by Title V, community college students need to complete coursework equivalent to Intermediate Algebra in order to obtain an Associate's Degree and/or to transfer to the California State University (CSU) System or University of California (UC) System [http://www.cccco.edu/Portals/.../guidelines\\_t5\\_chapter6part1\\_06252008.doc](http://www.cccco.edu/Portals/.../guidelines_t5_chapter6part1_06252008.doc). As a result of these changes in requirements, an overwhelming number of community college students now require math support resources and services because they lack the developmental skills to complete a course of intermediate algebra.

Many of these students lack the basic framework for mathematics, which begins in elementary school. The foundation of adding, subtracting, multiplying, dividing, and decimals, established in the elementary years, lays the ground-work for mathematical thinking and reasoning, including number value, order, word problems, space, and geometry. The California Department of Education (CDE) has created a K-12<sup>th</sup> grade mathematic framework for public schools to guide the curriculum development and

instruction that teachers provide in their efforts to ensure that all students meet or exceed current mathematics standards. Emphasis is placed on computational and procedural skills, conceptual understanding, and problem solving (California Department of Education, 2006). When students lack these basic mathematical skills they face multiple challenges, and will be under prepared for college and university systems that give them the opportunity to advance into careers based on attaining a college degree. The purpose of developing individualized strategies is to address a student's specific area(s) of difficulty, and to apply proven methods to their process and evaluation of learning. Learning strategies entail the fundamental steps of comprehension and cognitive functioning (Weinstein, 1988). Secondary and post-secondary students who struggle academically improve learning and cognitive engagement when they apply learning strategies. A positive correlation exists between students who use learning strategies and grade point average (Murray, 2000). Strategies can aid students throughout their academic curriculum and foster success, in education, future careers, and life.

#### Statement of the Problem

In 2008-2009, American River College (ARC) which is a prominent community college campus in Northern California, and its satellite campuses, had enrollment of approximately 38,000 students. In 2007-2008, enrollment was 33,821 (17,388 full-time equivalent) students. This represents an 11% increase of students in one single academic year. It is estimated that about 75% of first time students (first-time and first-time transfer) will need to enroll in developmental level English and/or math courses. These students represented over 50% of the ARC student population in Fall 2007 (Barr, 2010).

Due to the new graduation standards for math, students with learning disabilities are challenged in their requirement to complete Intermediate Algebra.

The learning disability staff utilizes handbooks for writing and English grammar as resources for individualized strategies. These accommodations benefit students with learning disabilities. Resources such as *You are Smarter Than You Think!* (1992), *Becoming a Master Student* (2006), and *Study Skills Strategies* (1994), enable students to develop critical thinking skills and to evaluate curriculum. These new skills allow the students to improve their academic performance. Individualized strategies for writing and English can be beneficial to students who have challenges with language and writing. Due to the success of individualized strategies for writing and English, the leadership and staff of the Learning Disability Department agree that it would be beneficial to develop a math strategies handbook for the Instructional Assistants (IAs) and Learning Disability Specialists (LDSs) to better meet the needs of students who struggle in mathematics.

The department of Disabled Students Programs Services (DSPS) staff believes it is paramount that IAs and LDSs become better equipped to provide greater services to students with learning disabilities. However, many IAs and LDSs in the department lack confidence in math. They find it difficult to support students in math because they feel inadequate in math themselves. A math strategies handbook is needed to equip staff, just as with English skill building, to better meet students' needs and to enhance the quality of education received at ARC.

### Purpose of the Project

The purpose of this project is to develop a handbook for math to be used as a reference tool to assist the IAs and LDSs who provide individualized strategies to students with learning disabilities. The handbook will outline a step-by-step approach in the application, reasoning, and problem solving of mathematics to better equip staff to support students. The focus of the project is to motivate and encourage IAs and LDSs who lack confidence in math and want math strategies tools to better serve their student population. The math strategies handbook will focus on concepts relating to, leaning styles, math vocabulary, and inductive reasoning. Also, it will provide tools on applying methods, and solve word problems. The math strategies handbook is designed to supplement math curriculum, foster independence in learning, and enhance learning by teaching math in a step-by-step process.

### Methodology

The methodology first will consist of a current review of math instructional strategy books to identify best practices for mathematics. Next, these strategies will be cross-referenced with learning strategies used by the learning disabled population. Then, mathematics concepts will be extracted from the currently used college math textbook and correlated with the learning strategies identified as best practices for the learning disabled population. Finally, this information will be compiled into a handbook and will be reviewed by IAs and LDSs in the department of Disabled Student Programs and Services at American River College.

## Definition of Terms

### *Disabled Students Programs and Services (DSPS)*

The goal of Disabled Students Programs and Services is to promote equal access to programs and facilities at American River College, thereby insuring that students with disabilities may participate fully in campus activities. The philosophy of DSPS is to encourage maximum independence and personal empowerment through a successful educational experience.

### *Learning Disabilities (LD)*

The Learning Disabilities (LD) Program provides learning disabilities assessment, accommodations and services. Study strategies and support are available to students individually, in groups, and in classrooms. In addition, the LD staff work with faculty to provide assistive technology for students. A learning disability is defined as a persistent condition of presumed neurological dysfunction, which may exist with other disabling conditions. This dysfunction continues despite instruction in standard classroom situations. Adults with learning disabilities, a heterogeneous group, are characterized as having:

- average to above average intellectual ability
- severe processing deficit(s)
- severe achievement-aptitude discrepancy(ies)
- measured average achievement for age in at least one instructional area.



### *Articulation Agreement*

The community colleges work together with the University of California, the California State University and private postsecondary colleges/universities to establish a course agreement to enable transfer students to receive credit for their academic programs.

### *Laws*

Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1990 (ADA) (Tincani, 2004) are laws which extend civil rights protection for people with disabilities in areas including employment in the public and private sectors, transportation, public accommodation, services provided by state and local government, and telecommunication relay services. These laws also require equal access to postsecondary education and employment opportunities.

### *Math 25 – Computational Arithmetic*

This course covers fundamentals of arithmetic with an emphasis on computational skills. Topic includes whole numbers, fractions, decimals, problem solving, and applications.

### *Math 32 – Pre-Algebra*

This course briefly reviews the fundamentals of arithmetic, including whole numbers, fractions, and decimals. Course content includes order of operations, signed numbers, concepts of variables, exponents, ratios and proportions, area/perimeter/volume of geometric figures, and solving equations.

*Math 100 – Elementary Algebra*

This course includes the fundamental concepts and operations of algebra with problem solving skills emphasized. Topics include properties of real numbers, linear equations and inequalities, integer exponents, polynomials, and factoring polynomials. Other topics include rational exponents and rational/radical expressions with associated equations. Additional topics include introducing the rectangular coordinate system focusing on graphs and equations of lines, systems of linear equations/inequalities, and solving quadratic equations.

*Math 120 – Intermediate Algebra*

This course reviews and extends the concepts of elementary algebra with an emphasis on problem solving. Topics which are reviewed and extended include linear and quadratic equations, factoring polynomials, rational expressions, exponents, radicals, graphing, and systems of equations. New topics include graphs and their translations and reflections, functions, exponential and logarithmic functions, graphs of quadratic and polynomial functions, nonlinear systems of equations, polynomial and rational inequalities, and an introduction to conic sections.

*Math Multimedia Learning Center (MMLC)*

The MMLC is an independent study option (Math 1000) for students taking a class in Arithmetic (Math 25), Pre-algebra (Math 32), Beginning Algebra (Math 100), or Intermediate Algebra (Math 120). The center's classes are self-paced. Students may enroll for up to two semesters or may complete the study as quickly as they are able. The courses are computer-based and students can do some of their work at home on their own

computers. Students in the MMLC can enroll until the 11<sup>th</sup> week of school for fall and spring semesters. The program is also open during the summer. Free tutoring also is provided at the center.

### *Learning Resource Center (LRC)*

The Learning Resource Center (LRC) is a professionally-staffed facility offering students a personal approach to academic success through classes, independent study, individualized tutoring and alternative modes of instruction. The LRC houses the Reading Center, the Writing Center, and the English as a Second Language (ESL) Center, the Tutoring Center, and the Athlete Academic Services. In addition, the LRC houses the WAC (Writing Across the Curriculum) program, the RAD (Reading Across the Disciplines) program, the Beacon Program, and the Foreign Language program.

### *BEACON*

The BEACON Program provides group tutoring for students in various courses. Groups meet every week with a tutor who has successfully completed the course with the same instructor. The tutor works with the instructor and is prepared to address the students' needs. This service is provided free to students and is available Fall, Spring, and Summer semesters. With instructor approval, students who received an "A" or "B" in a course can become a Beacon tutor. Tutors are salaried and must enroll in the Beacon Tutor Training Course INDIS 321 at the beginning of the semester.

### *Developmental Education*

The term refers to remedial, basic skills, and development education in English, Math, and ESL (English as a second Language) that fall below transfer level.

### Limitations of the Project

The math strategies handbook will function as an assistive or instructional tool to aid IAs and LDSs in implementing individualized learning strategies with students who are challenged in math. Students with a learning disability will attain knowledge about their leaning styles, build confidence, evaluate mathematical patterns, and interpret abstract concepts. Although the project addressed the department's needs regarding staff success, there are other areas for future research. The handbook is specifically limited to the staff at American River College in the Learning Disability Department. The math strategy handbook is the only developed handbook within the department. Some constraints of the project do not show staff success and student academic performance outcomes within the given semester. Another restraint of this project is due to the bias options of the researcher who is a participant- observer as an Instructional Assistant at America River College in the Learning Disability Department.

### Organization of the Project

In this project, Chapter one includes the following: Statement of the Problem, Purpose the Problem, Definition of Terms, Delimitation and Organization of the Project. Chapter two provides a review of the literature. It will include pertinent information on students who are underprepared for mathematics at the college and university level, influx of students learning disabilities entering post secondary education systems, the lack of self-esteem, and individualized learning strategies. Chapter three describes the procedure of the handbook, collaboration of data, and data analysis of learning strategies that are most effective (best practices). Chapter four will contain the summary of the

project, draws conclusion, and implications for the use of the math strategies handbook and further research.

## Chapter 2

### REVIEW OF THE LITERATURE

#### Introduction

The Algebra Math Handbook project will provide step-by-step applications and implementation for learning strategies essential to the delivery for educational instruction of Instructional Assistances (IAs) and Learning Disability Specialists (LDSs) to students with learning disabilities so they can better meet their needs. It will foster confidence, encourage one's ability, and create positive attitudes towards mathematics. The following chapter will discuss how learning disabled students are underprepared for college level mathematics and lack the fundamental foundations for arithmetic. The literature will review the increasing number of disabled students who are entering college and university systems and explain the definition of a disability and the laws which protect disabled students. The influence of low self-esteem and its effects on mathematic achievement will be examined. Mathematical concepts and how it is learned will segue in to the effectiveness of learning strategies for disabled students.

#### *Students who are Underprepared for College*

More than half of the students who do manage to graduate from high school, and more than two-thirds of all students who start high school, do not graduate with the minimal requirements needed to apply to a four-year college or university. A study by Greene & Foster (2003) estimated the percentage of students in the public high school who possess the minimum qualifications for applying to four-year colleges. The data collected comes from a large national study performed by the U.S. Department of

Education to estimate college readiness rates. The results concluded that only 70% of all students in public high schools graduate, and only 32% of all students who leave high school qualified to attend four-year colleges. The graduation rate for white students was 72%; for Asian students; 79%; and for American Indian students, 54%, black students was 51%; and Hispanic students 52%. Graduation rates in the Northeast (73%) and Midwest (77%) were higher than the overall national figure, while graduation rates in the South (65%) and West (69%) were lower. The Northeast and the Midwest had the same college readiness rate as the nation overall (32%) while the South had a higher rate (38%) and the West had a lower rate (25%). Other findings estimate that there were about 1,299,000 college-ready 18-year-olds in 2000, and the actual number of persons entering college for the first time in that year was about 1,341,000. This indicates that there is not a large population of college-ready graduates who are prevented from actually attending college. The data suggest that the main reason these groups are underrepresented in college admissions is that students are not acquiring college ready skills in the K-12 education system.

In some cases, academic deficiencies are so severe that colleges choose to expel students. In an article by Trounson (2002), the California State University system in 1996 set a goal that only 10% of incoming freshmen by 2007 would require remedial classes. In 2002, 46% of incoming freshmen needed remedial classes. One year prior, the California State University system ousted 2,200 students (7% of the freshmen class) who failed to master basic English and math after one year of remedial help. A \$9-million program enabled California State University faculty members to reach out to 172 high

schools around the state to better prepare students entering the university system, though the effectiveness of the program was not measured. In 2001, California State University system required nearly one-third of first-year students to take a remedial course in reading, writing, or mathematics (National Center for Education Statistics (NCES), 2003).

American River College (ARC), a community college in Northern California, estimated that 75% of first-time freshmen, first-time transfers, and returning students (13,801) will need to enroll in developmental /remedial level Math and /or English courses. These students represented over 50% of the ARC academic student population in the Fall 2007 semester (Barr, 2008). Barr also states that from 2005 to 2009, American River College records indicate that the majority of incoming students who took the placement test for the first time tested below transfer level for English, math, and ESL. Yet these students enroll in transfer level courses across the campus during their first term.

A breakdown of the American River College data over the last five years is as follows: 26.2% of students were at transfer level compared to 73.8% who performed below transfer level courses in English Writing; 27.6% of students were at transfer level compared to 72.4% below transfer level courses in English Reading; 8.1% of students were at transfer level compared to 91.9% below transfer level courses in ESL Reading; and 15.5% compared to 84.5% below transfer level courses in Math (Barr, 2010).

There are many factors as to why students entering college and university systems are underprepared. Unfortunately, many students do not develop effective learning strategies



unless they receive explicit instruction and the opportunity to apply these skills (Weinstein, et al., 2000). Hawkes & Savage (2000) refer to the serious decline in students' mastery of basic mathematical skills and level of preparation for mathematics-based degree courses. Acute problems impact colleges and university systems, including an increasingly diverse student population, unstable funding, limited resources, and students with disabilities who are underprepared for college level study.

*The Increase of Learning Disabled Students on College and University Campuses*

The growth in the number of students with learning disabilities (LD) attending colleges and universities has been documented the world over (e.g., Heiman & Precel, 2003; Henderson, 2001; Higher Education Statistics Agency, 2003; National Center for Education Statistics, 1999). Public attitudes have changed profoundly towards individuals with disabilities and the need for colleges and universities systems to comply with laws that protect the right of individuals with disabilities to attend institutions of higher learning. In the United States, Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act (ADA) protect the rights of these students, guaranteeing them the right to reasonable accommodation both in the admission's process and once they have matriculated (Vogel, Fresko, & Wertheim, 2007). Similarly, Britain's 1995 Disability Discrimination Act, the Dearing Report of 1997, and the Special Educational Needs and Disability Act (SENDA) of 2001 has impacted eligibility requirements and has led to a growth in the number of students with LD who attended intuitions of higher learning (Stacey & Singleton, 2003). In Canada, the admission of students has been voluntary in

nature but has seen an escalation in the number of incoming students with LD entering educational institutions (Bat-Hayim & Wilchesky, 2003).

Research by Henderson (2001) provided statistical data describing students who reported disabilities and enrolled in Fall 2000 as full-time freshmen at public and independent four-year colleges and universities. Since 1996, a national college survey has been administered to freshmen across the nation by the Cooperative Institutional Research Program. The survey was administered to 269,413 students at baccalaureate four-year colleges and universities. Data collected gives a profile of first-time freshmen students at the beginning of their college experience. The survey results revealed between 1988 and 2000, learning disabilities were the fastest growing category reported of disabilities among students. By 2000, two in five freshmen with disabilities (40%) cited a learning disability compared to only 16% in 1988. The number of freshmen with learning disabilities rose substantially during this 12 year period (Henderson, 2001).

#### *Under The Law Defining a Disability and Learning Disability*

Many students enter college underprepared and lacking the basic skills for college level mathematics. Experts estimate that 6 to 10% of the school-aged population in the United States is learning disabled and nearly 40% of the children enrolled in the nation's special education classes suffer from a learning disability. The Foundation for Children with Learning Disabilities estimates that there are 6 million adults with learning disabilities live within the USA (Child Development Institute, 2007).

The Americans with Disabilities Act (ADA) of 1990, a law which provides programs and supports students with learning disabilities at the postsecondary level,

describes a disability as anyone with a physical or mental impairment that substantially limits one or more major life activities, such as caring for one's self, performing manual tasks, walking, seeing, hearing, speaking, breathing, learning, and working. In addition to these people who have visible disabilities—persons who are blind, deaf, or use a wheelchair—the definition includes people with a whole range of invisible disabilities which include psychological problems, learning disabilities, or some chronic health impairment, such as epilepsy, diabetes, arthritis, cancer, cardiac problems, HIV/AIDS, and more (ADA, 2008) <http://www.ada.gov/index.html>.

The Individuals with Disabilities Education Act (IDEA), initially enacted in 1975, provides special education and related services for children with disabilities. It defines a specific learning disability as a disorder in one or more of the basic psychological processes involved in understanding or using language, spoken or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell or to do mathematical calculations. The term includes such conditions as perceptual handicaps, brain injury, minimal brain dysfunction, dyslexia and developmental aphasia. The term does not include children who have learning problems which are primarily the result of visual, hearing, motor handicaps, mental retardation, emotional disturbance, or of environmental, cultural, or economic disadvantage (IDEA, 2004) <http://idea.ed.gov/explore/home>.

### *Self-esteem, Attitudes and Academic Performance*

Individuals possess a belief system that enables them to exercise control over their thoughts, feelings, and actions. This belief system consists of cognitive affective

structures that allow individuals to regulate their own behavior and engage in self-reflection. As such, the belief system provides individuals with the capability to influence their environments and determine their own actions (Bandura, 1986).

According to Marsh (1993), teachers are significant others for most students, and thus appraisals received from teachers may have a strong impact on students' self-concept of ability and self-esteem.

The purpose of a study by Groinick & Ryan (1990) was to compare learning disabled (LD) students with non-learning disabled (NLD) students and low achieving students on self-concept dimensions. They concluded that children with LD perceive themselves to be less academically capable than other students with the same IQ who are achievers. They found no differences in academic self-perceptions compared with the low achievers. Poor self-regard and self-worth, moreover, were also found in other low-achieving students due to their continuing struggle with learning. All children who find themselves in an academic failure cycle at school tend to develop poor self-worth that ultimately leads to lower self-regard. If children do not think they have much value, they will naturally feel less able to like themselves.

Quigley (1992-1997) suggests that many adult literacy educators believe that their learners have lived lives of failure resulting from low self-esteem. Until the deficits are addressed and corrected, the cycle of continuous failure will repeat. Kerka (1988) supports this view by stating that educationally disadvantaged adults who had a negative past schooling experience are more likely to lack self-esteem, have negative attitudes towards education, and lack mastery of basic skills. A recent study by Lipnevich &

Beder (2007) investigated 200 adult literacy education learners' self-esteem and examined the relationship between self-esteem and measures of achievement in math and reading. The data indicated that both the reading and math scores were significantly and positively correlated with the scores on academic and global self-esteem. High correlation between the academic and general self-esteem scales indicates that learners who have low academic self-esteem tend to have low global self-esteem, and vice versa. This suggests that for adult literacy learners, the specific domain of academic self-esteem significantly influences their global self-esteem and vice versa.

#### *Mathematical Concepts and How They Are Learned*

Cognitive development is defined as development of the ability to think and reason (University of Chicago Comer Children's Hospital, 2006). The framework for mathematics begins in the early stages of life and building blocks form links of understanding throughout adolescence, teenage years, and adulthood. Heaton (2000) a professor of mathematics, stated that:

...learning math is learning to communicate a mathematical ideal or interpret the mathematical representations of others, through the use of language, diagrams, pictures, manipulative, and other tools to aid communication ... Learning takes place as students actively construct meaning for themselves by connecting new ideas with previous understanding. (p. 54)

Jean Piaget also believed students learn by fitting new information together with what they already know (Davison, 2006). Moreover, he describes Piaget's Theory of Cognitive Development which consists of four stages. The first stage is sensor motor where

intelligence manifests itself through motor activities. Most of the knowledge acquired during this stage is through physical activity. The second stage is called the preoperational (egocentric stage). Children believe that everyone thinks exactly as they do. Children begin to use symbolism, oral language, memory, and imagination. The author's primary focal point is on Piaget's third stage of cognitive development which is concrete operation. Between the ages of seven and eleven children experience a dramatic change in the way they think. Thinking becomes-less egocentric and more logical. Reversibility, the ability to perform a mental operation and then reverse one's thinking to return to the starting point, manifests itself prominently during this stage (Slavin, 2003).

The final step in Piaget's Cognitive Development Theory as described by the author is the operation stage. This stage provides those who attain it with the ability to master abstract thought and use symbols. Children do not simply absorb information for the world; they are not simply shaped by the environment. Rather, children actively construct concepts, understanding, strategies, and modes of thought (Noddings, 1990). According to Piaget, children learn because their minds are biologically designed to develop concepts and modes of thought useful for adaptation to the environment (Piaget, 1952b).

The California Department of Education most recent edition of the Mathematics Framework for California Public Schools (Kindergarten through Grade Twelve, 2006) explains:

When students delve deeply into mathematics, they gain not only conceptual understanding of mathematical principles but also knowledge of and experience with pure reasoning. One of the most important goals of mathematics is to teach

students logical reasoning. Mathematical reasoning and conceptual understanding are not separate from content; they are intrinsic to the mathematical discipline that students master at the more advanced levels. (p. 3)

The authors further note that students who do not have a deep understanding of mathematics suspect that mathematics is just a jumble of unrelated procedures and incomprehensible formulas. For example, children who do not understand the basic counting concepts view counting as rote, a mechanical activity. In contrast, children with a good conceptual understanding of counting understand that items can be counted in any order – starting from right to left, skipping around, and so forth – as long as each item is counted only once (Gelman & Meck, 1983).

For each level of mathematics, a specific set of basic computational and procedural skills must be learned. Students need to memorize the math facts of addition and multiplication of one-digit numbers and their corresponding subtraction and division facts. The ability to retrieve these facts automatically from long-term memory makes solving of more complex problems, such as multi-step problems that involve basic arithmetic, quicker and less likely to result in errors (Geary & Widaman, 1992). As students progress through elementary school, middle school, and high school, they should become proficient in the following skills: solving addition, subtraction, multiplication, and division problems, finding equivalencies, performing operation, measuring, perimeters and areas, interpreting graphs, means and medians, using scientific notation, basic geometry, equation of a line, and solving linear equations and systems. This affords the capacity to complete intricate

problems in subjects such as Algebra. The way children learn and mentally grow plays a central role in their learning processes and abilities (Davison, 2006).

Making meaning out of mathematics is a social activity. The reforms are based on the belief that “students’ learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas” (NCTM, 1989, p. 58). An article for the Learning Theories Knowledgebase (2010) refers to Russian psychologist Vygotsky’s Social Development Theory and argues that social interaction precedes development; consciousness and cognition are the end products of socialization and social behavior. Russian psychologist Vygotsky lived from 1896 until 1934, and believed that social interaction plays a fundamental role in the process of cognitive development. In contrast to Jean Piaget’s understanding of child development (in which development necessarily precedes learning), Vygotsky felt social learning precedes development. He states: “Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological).” Vygotsky’s next developmental stage is the More Knowledgeable Other (MKO). It refers to anyone who has a better understanding or a higher ability level than the learner, with respect to a particular task, process, or concept.

The MKO is normally thought of as being a teacher, coach, or older adult, but the MKO could also be peers, a younger person, or even computers. The last developmental stage is the Zone of Proximal Development (ZPD). The ZPD is the distance between a student’s ability to perform a task under adult guidance and/or with peer collaboration and



the student's ability to solve the problem independently <http://www.learning-theories.com/vygotskys-social-learning-theory.html>. Vygotsky focused on the connection between people and the sociocultural context in which they act and interact in shared experiences (Crawford, 1996).

Davison (2006) further explains Vygotsky's pioneering work in developmental psychology. The author claims Vygotsky's idea of a zone of proximal development is the second aspect of his cognitive theory. "Zone of proximal development" (ZPD) is the range of tasks that are too difficult for the child to master alone, but can be learned with guidance and assistance from adults or more skilled children. The ZPD captures the child's cognitive skills that are in the process of maturing and can be accomplished only with the assistance of a more-skilled person.

Scaffolding involves encouragement and assistance in the form of advice and suggestions to aid a child in mastering a new concept. By using hints and pointers from teachers, parents, and peers who have already grasped the desired concept, children are able to form their own path towards a solution. Eventually they become able to self-regulate, or think and solve problems without the help of others (Slavin, 2003). Davison concludes that cognitive development plays a key role in learning and thinking methods of children.

The key ideas of Piaget and Vygotsky's theories differ in that Piaget believed that intelligence came from action. He held that children learn through interaction with their surroundings and the learning takes place after development. Alternatively, Vygotsky believed that learning happens before development can occur and that children learn because of history and symbolism. He also believed that children value input from their

surroundings and from others. These two developmental theories of mathematical learning are applied to mathematical curriculum that is used today.

A comparative analysis conducted by Sood & Jitendra (2007) compared number sense instruction in three first-grade traditional mathematics textbooks and one reform-based textbook (Everyday Mathematics [EM]). Textbooks were evaluated by their adherence to principles of effective instruction. Results indicated that EM and traditional textbooks differed with respect to their adherence to the two big ideas in number sense: Traditional textbooks included more opportunities for number relationship tasks, in contrast, EM emphasized more real-world connections. EM scored better than traditional textbooks in promoting relational understanding and integrating spatial relationship tasks with other more complex skills. Traditional textbooks were more direct and gave explicit feedback and more opportunities to practice number sense skills. However, EM draws more attention to concrete, semi-concrete, and symbolic representations. EM provides more opportunities for students to practice exercises and problem solving using more than one representation, devoted more lessons to activities, and utilized a variety of models to develop number sense.

Cognitive development plays a key role in the learning and thinking methods of children. The Comprehensive School Mathematics Program (CSMP) is a curriculum oriented to functions and is designed to teach mathematics as a problem solving activity rather than just teaching arithmetic skills. From this foundation, the expectation is that in later grades students will be able to do the type of abstract generalizing required in the explicit study of algebra [www.ed.gov/pubs/triedandtrue/-compr.html](http://www.ed.gov/pubs/triedandtrue/-compr.html). The primary goal of

the Comprehensive School Mathematics Program is to provide a complete kindergarten through sixth grade mathematics program which develops a broad and balanced range of skills regardless of students' ability levels--a program that will actively involve students in the world of mathematics, not simply drill them in the techniques of arithmetic. As a result, students will be able to understand the content and applications, develop techniques for learning them, and use their mathematics to solve problems. The content is completely sequenced in a spiral form so that a student is brought into contact with each area of content continuously throughout the program. Students work through repeated exposures to the content, building interlocking experiences of increasing sophistication

[www2.ed.gov/pubs/triedandtrue/compr.html](http://www2.ed.gov/pubs/triedandtrue/compr.html).

Moses (2000), a mathematics educator and researcher, argues that algebra is about indentifying patterns and expressing these patterns in a logical fashion. She believes mathematics deals with general statements of relations, using letters and to the symbol to represent specific sets of numbers. She further says, that students must learn to collect and organize data so they can recognize patterns in the data then express what they observed. When teachers take this approach, they teach students to think mathematically and bridge the gap from arithmetic to algebra. For example, students are asked to create rectangles with an even number of colored tiles, they notice when they apply this method that one dimension could always be two.

Students concluded that every even number could be written as two times some other number-as a small step away from an even number being  $2x$ . Moses continues to states that algebraic thinking is the vehicle for exploring the world regularities. A young

child may investigate the table of basic addition facts and note symmetry across diagonals, thus recognizing the commutative property. While and older children may note that the sum of the first  $n$  odd natural numbers seems to generate square numbers. The representations of these relationships may change according to the maturity of the students, but we must encourage algebraic thinking at every level to instill success.

*How Students with Learning Disabilities (LD) Learn Mathematical Concepts*

A report by Ginsburg (1989) asserts that researchers usually take constructivist and interactionist points of view when addressing children's intellectual development, which drawing heavily on the works of both Piaget and Vygotsky. The child's mind always develops in both a physical and social environment. The goal is to understand the cognitive processes and other factors underlying the academic failure of children with learning disabilities. For children with learning disabilities (LD), connecting basic mathematical concepts is difficult when there is an interference with their cognitive processes.

One of the most consistent research finding is that children with LD have a particularly difficult time remembering basic number facts (Geary, 1993). Approximately 6% of elementary school and junior high school children are diagnosed with a mathematics learning disorder, compared to the approximately 5% with a reading disability (Badian, 1983; Geary, 1993). Learning to count involves one set of cognitive skills (i.e., initially memorizing the numbers 1 to 12, and then detecting rules underlying the language), whereas appreciating the notion of equivalence requires other cognitive operations (perhaps something like Piaget's [1952a] concrete operation "reversibility").

According to Wetzel (2009), math learning disabilities are often caused by visual-spatial-motor disorganization. As a result, these learning disabled students experience a weak understanding or lack a comprehension of concepts, have very poor number sense skills, as well as difficulty with pictorial representation, poorly controlled handwriting, and confusion with arrangements of numerals and signs on textbook/workbook pages. Children encounter trouble in connecting knowledge base information to the more formal procedures, language, and symbolic notation system of school math. Students need many repeated experiences and many varieties of concrete materials to make these connections strong and stable (Garnett, 1998).

Recent developmental theory stresses that cognitive development always takes place within a social and cultural setting (Resnick et al., 1991). Vygotsky believed that since children learn much through interaction, curricula should be designed to emphasize interaction between learners and learning tasks and these social cognition learning models assert that culture is the prime determinant of individual development <http://www.funderstanding.com/-content/vygotsky-and-social-cognition>.

### *Mathematical Training for Teachers*

The importance of teacher training sets the stage for quality education which invites students to learn and become active participants. An article by Ballheim (2000) looks at the responses from readers (teachers and educators) from a questionnaire about in-service training on mathematics. Four questions were selected and distributed among the readers and the responses than posted. The data from the responses were then reviewed by the Nation Council of teacher of Mathematics to learn how to provide

quality teaching and educational support to teacher in-service training sessions on mathematics. In response to the questioner, most of the readers strongly felt that short-term in-service sessions were valuable, less commitment time was needed to attend sessions, and the core of information was presented right away. Many readers believed that the in-service education was beneficial and that further staff development sessions would be useful. Most readers agreed that the two-hour-long workshops were optimal and allow time for hands activities with time for questions.

Readers believed that short in-service sessions that were conducted by teachers who were competent in their fields were extremely helpful in assisting well-informed teachers in staying on the cutting edge of mathematics teaching. This was unanimously agreed upon by these teachers who had more experience and had greater expertise about content and instructional strategies. The readers believed that all teachers should be learning new content and upgrading their own skills all the time. Although most readers stated that mathematical content should consist of computers, graphing calculators, innovative teaching methods, and statistics which are applicable to the real world. Finally, most readers felt the teachers should conceptually understand the framework for effective teaching and learning and be confident in relearning the mathematics as they help their students understand it.

#### *The Effectiveness of Learning Strategies for Student with Disabilities*

Students must have skills to succeed academically in the use of learning, finding the main ideas of course content comprehension, notation, and logical problem solving (Rachal, Daigle, & Rachal, 2007). Unfortunately, many students entering college have

little understanding of what skills are needed for learning and have not developed effective learning strategies. Weinstein (1988) and Weinstein, Husman, & Dierking (2000) argue that:

Learning strategies entail the fundamental steps of comprehension and cognitive functioning. They refer to methods and techniques used by students to improve learning. These techniques include asking questions, taking notes, developing study schedules, using SQ3R, which are essential to the learning process. It is the applications of these techniques that facilitate the enhancement of knowledge retrieval and integration and are considered to be learning strategies. Such strategies enable the learner to develop procedures for performing higher-level mental procedures (p. 192)

The research conducted by Rachal, Daigle, & Rachal (2007) focused on undergraduate students reported learning difficulties and the use of learning strategies across the curriculum. The research concluded the majority of students who reported difficulties in reading (comprehend text while studying), writing (putting thoughts in to words), and mathematics (solving complex math problems in algebra). The data further implies that students should receive explicit instruction in the use of learning strategies during first year orientation or study skills courses. Students then would be provided with opportunities to apply general as well as content specific learning strategies. This would ensure a solid foundation in how to improve their ability to process information more effectively and continued motivation for learning. Secondary and post-secondary students who struggle academically improve learning and are more cognitively engaged

when they apply strategies. Students who lack problem-solving strategies generally need explicit instruction in specific cognitive strategies (e.g., visualization, verbal rehearsal, paraphrasing, summarizing, estimating) to facilitate their reading, understanding, executing, and evaluating of problems (Wong, 1992).

Learning strategies are positively correlated with student achievement (Murray, 2000). A study conducted by Corral & Antia (2002) show students with learning disabilities who struggle with mathematics can turn the failure cycle into a success cycle through learning strategies and positive attributions. The Attribution theory states that when a person feels success is possible, he or she is likely to exert great effort, persist for a longer period of time, and attribute a greater proportion of success to the effort exerted contrary to someone who does not expect success (Carr, Borkowski, & Maxwell, 1991; Deshler, Schumaker, & Lenz, 1984; Garner, 1990, Yasutake, Bryan, & Dohrn, 1996). According to Corral & Antia (2002), successful learning strategies consist of combining the following eight steps:

1. Model the learning strategy, verbalizing each step.
2. With the students, discuss the strategy steps, which you have written down and kept visible for reference.
3. Discuss the strategy's rationale and value.
4. With the students, simultaneously apply the strategy steps to a new problem while stating the steps (referring to the visual cues, if necessary).
5. Encourage the student to apply the strategy to another problem while stating the steps; watch the student and provide corrective feedback as needed.
6. Ask the student to apply the strategy to another problem without stating the steps; provide



corrective feedback as needed. 7. Encourage the student to apply the strategy to a few math problems independently; check for correctness. 8. Ask the student to state the strategy steps from memory and show how he or she used them with the six steps to positive attribution in math which are as follows: 1. Model correct strategy application, stressing the strategy's value to students. 2. Model positive attribution statements often as the kind of self-talk that successful math students use. 3. Model positive self-talk when discovering errors in own work (or create intentional errors to discuss). 4. Allow students to periodically reflect on class math tasks and reason for their success or failure through the use of self reports or journals. 5. Encourage students to keep personal records of the positive attribution statements they make when working. 6. Encourage students to set specific goals and use goal statements by doing the following: a) keeping a list of individual goals and reading them silently before beginning the days' assignment, b) Self-checking, test-taking goals, such as "Check all basic operations when finished," or "Read the directions twice," c) Using positive attribution statements for test-taking, such as "I have done problems like this before so I can think positively about these," or "If I use my strategies carefully, I will probably be successful (p. 45)

The study reported that the student used 24 more positive statements and only 3 negative self-statements. In turn, when the student felt unsuccessful, he perceived the level of task difficulty as more important than the amount of effort he exerted.

Conversely, when he felt successful he perceived the level of task difficulty as less important, and the amount of effort exerted as more important. The study also found that the student persisted longer and more frequently with challenging math problems. Students begin to make the connection between effort and success by experiencing it firsthand they see that learning strategies do “pay off,” and put them in greater control of their own learning. Corral and Antia (2002) conclude that to successfully combine strategy instruction with attribution re-shaping, models of the strategy and self-statements encourage reflection and help students set goals and self-monitor them. Learning strategies used in kindergarten through high school will be applicable in high levels of education as students with learning disabilities gain access to academia.

Students with disabilities in general, and those with learning disabilities (LD) at the middle school level, often have difficulty meeting academic content standards and passing state assessments. According to Maccini and Gagnon (2005), some key features that make learning strategies effective for students with LD are:

- (a) Memory devices to help students remember the strategy (e.g., a First Letter Mnemonic, which is created by forming a word from the beginning letters of other words);
- (b) Strategy steps that use familiar words stated simply and concisely and begin with action verbs to facilitate student involvement (e.g., read the problem carefully);
- (c) Strategy steps that are sequenced appropriately (i.e., students are cued to read the word problem carefully prior to solving the problem) and lead to the desired outcome (i.e., successfully solving a math problem);
- (d) Strategy steps that use prompts to get students to use cognitive abilities (i.e., the

critical steps needed in solving a problem); and (e) Metacognitive strategies that use prompts for monitoring problem solving performance: “Did I check my answer?” (p. 2).

When students have a positive frame of reference of their ability and a positive attitude toward mathematics, they are more likely to approach instruction with an optimistic view and will become committed to learning (Wong, 1994). These factors are important in effective strategy instruction (Ellis, 1993). A study by Wetzel (2009) looks at the math learning strategies designed to help students grasp concepts. Wetzel believes teaching math to learning disabled students can be challenging because they have difficulty bridging the gap between informal math knowledge and formal school math concepts. Structured, concrete, and hands-on materials are important in trying these links between concepts.

Wetzel’s strategy not only applies to elementary grades, it is also vital during concept development states of higher-level math. Teaching math is compounded by the common difficulties special needs students have concerning proficient recall of basic arithmetic facts, along with consistency in written computation. When these problems are accompanied by a need for strong conceptual grasp of mathematical and spatial relations, it is imperative the students are not focusing only on remediation computation. The Wetzel concludes that teaching math strategies should include the use of pictures or graphics for conveying concepts, constructing verbal versions of math ideas, and using concrete material in math activities.

Research evidence reveals that students who use concrete materials actually develop more precise, comprehensive mental representations (Garnett, 1998). As a result, students often show more motivation and on-task behavior, may better understand mathematical ideas, and may better apply these to life situations. The author concludes that structured, concrete materials have been beneficially used to develop concepts and to clarify early number relations. The concrete materials also includes place value, computation, fractions, decimals, measurement, geometry, money, percentage, number bases story problems, probability, statistics, and algebra.

Some students with learning disabilities are hampered by the language aspect of math, so it is important that teachers slow down the pace of their delivery, maintaining normal timing of phrases, and give information in discrete segments. Such slowed down “chunking” of verbal information is important when asking questions, giving directions, presenting concepts and offering explanations. Garnett further expresses that it is crucial for students to verbalize what they are doing. Students with language confusion need to demonstrate with concrete material and explain what they are doing at all ages and all levels of math. Furthermore, if students know the strategy and are given appropriate tasks and enough time to practice using the strategy within the mathematical context (e.g., mathematical problem solving, narrative composition, expository reading), then they can frequently apply the strategy properly (Pressley & Associates, 1990).

At the community college level, learning strategies can be effective at improving mathematical performance when IAs and LDSs provide mathematical instructions that are tailored to meet the need of a college student with a learning disability. The DSPS

department can provide ample instructional scaffolding with systematic model of active thinking, encourage interactive application, and promote active involvement in the learning process by students. We can conclude from the literature that a student that who has a command of the terminology and visual strategy steps can apply positive attribution steps which enhance confidence that lead to math success.

## Chapter 3

## METHODOLOGY

An Algebra Handbook: Instructional Strategies for Assisting College Students  
With Learning Disabilities

*Organization*

The intent of the Algebra Handbook is to provide a reference tool to assist Instructional Assistants (IAs) and Learning Disability Specialists (LDSs) who give math strategies to students with learning disabilities at the postsecondary level. This reference tool will introduce mathematical concepts, give related practical step by step strategies, and illustrate diagrams that provide concrete visual aids. The format of the handbook provides a table of contents, introduction, mathematical strategies, a table of mathematical terms and symbols, resources and note section.

The first step to creating the handbook began with a comparative analysis between the current Algebra textbooks used at American River College to a teacher's edition of Algebra to verify that the content are similar. The materials used were: *Algebra Structure and Method (Teacher's Edition)* 1992, *Data, Equations, and Graphs (Make Sense of Elementary Algebra)* 1971, *Intermediate Algebra (Eight Edition)* 2000, and *Intermediate Algebra (Models, Function, and Graphs)* 1997. The teacher's edition of Algebra was chosen as the foundation for this handbook because it provides teacher instructional strategies for each mathematical concept within every section of the textbook.

The second step involved was compiling a list of strategies and then grouping them into similar categories. These strategies were identified in the review of the literature chapter. For the purposes of this project, the handbook consists of the following categories: concept, strategy, visual aid and references.

For the third step, the researcher looked through the college algebra math book and matched teaching strategies from the teacher's edition to correspond with the units listed in the textbook. The determining factors of choosing a strategy for the handbook were based on the following:

- 1) met with the staff to receive their feedback on the handbook regarding current math challenges faced by students
- 2) strategies were modeled in a simplified step by step process and visual illustrations were presented when appropriate
- 3) precision of concise methodical explanations of instruction on how to apply concepts for solving equations
- 4) detailed illustrations were given as examples on how to use the instructional strategies.

The fourth step involved creating the layout of the instructional strategies. A four-column format was selected to help instructional assistants follow the step-by-step strategy with a visual image and/or example of a concrete problem that could easily be relayed to the students during the tutoring session.

The fifth step involved creating an introduction to the handbook. It contains information regarding the importance of student's learning styles and types of learning. Information on best questioning practices was also provided. Mathematic terms and symbols are given to connect conceptual understanding and meaning of algebraic equations. Visual illustrations were provided to demonstrate relational thinking and

abstract concepts. Also all illustrations and visual aids were prepared by the researcher within chapter three. Resources gave additional information on diagrams, exercises, and other innovative strategies. The notes section will allowed staff to incorporate their own ideas and develop new strategies that the department of Disabled Students Program Services has found successful. The handbook will include any other pertinent information that could be useful to the IAs and LDSs who provide instructional strategies to students with learning disabilities.

The sixth step included putting the contents of the handbook on card stock paper with tabbed sub sections, then placing all of the sections in a one and one-half inch binder in the following order: title page, table of contents, instructional strategies, resources, manipulative diagrams and notes. Finally, a training session for IAs and LDSs will take place prior to Fall of 2010 to demonstrate the use of the Algebra Handbook.



## Chapter 4

### CONCLUSION

The algebra handbook project was designed to be a reference tool to assist Instructional Assistants (IAs) and Learning Disability Specialists (LDSs) who provide instructional strategies for students with learning disabilities. The algebra handbook is comprised of fundamental step- by-step strategies, mathematical concepts, and illustrated diagrams used for the visual sequencing. Mathematical terms and symbols were used to present a logical understand towards solving equations. By providing additional resources on how students learn, it is anticipated that handbook users will have insightful support on how to approach students learning needs. The inclusion of accessible online websites offers further strategies and mathematical concepts.

Upon completion of the project, the researcher gathered feedback on the usefulness and content of the handbook from IAs and LDSs at American River College as well as from math professors from other community colleges. While they all endorsed the finished handbook, they also suggested expanding it. The researcher interprets this as IAs and LDSs having a need for even more supportive materials. Based on the feedback, the researcher would recommend that future editions of the handbook needs to include a broader resource base and more examples of helping strategies. It also suggests the possible lack of comfort of IAs and LDSs in terms of mathematics.

In the development of the handbook, the researcher met with the department staff once during the initial process of creating the handbook to gather input regarding its content and to discuss current challenges faced by students with learning disabilities. The

staff mentioned that more step-by-step math strategies would be beneficial to them. However, the researcher did not meet with the IAs and LDSs and it was the IAs and LDSs who expressed interest in expanding the number of step-by-step math strategies they requested to be included the handbook, not the administrative staff. For future projects, the researcher recommends that the first step of the entire process begin with a staff survey that includes the IAs and the LDSs, and focuses on their own experience with mathematics, level of completion in math, confidence level in math, and challenges in math. With this information, the handbook would have been developed on the premise of the data gathered from the survey to include the perspective of the IAs and LDSs as well as the administrative staff and the students whom they serve. This would address the math anxiety of the IAs and the LDSs who do the direct instruction.

The purpose of this project was to develop an algebra handbook. A review of this handbook's first use by the staff at American River Community College may have provided the researcher feedback on the effectiveness of the instructional strategies when serving students with learning disabilities in the area of algebra. This might be a starting point for future researchers who might choose to improve the handbook.

#### Future Research

There are several areas of study that would benefit from the creation of this handbook. For instance, the design of a longitudinal study on student academic performance in mathematics and how the strategies were used over a period of time could illustrate the success of the handbook's application for serving students with learning disabilities. Such a study may also uncover areas for further development and revision of

the handbook. Another area of study would be to measure the academic achievement of students with learning disabilities who successfully completed the next level of math after taking algebra with instructional support from the DSPS department with a staff that used this handbook.

Overall, the development of this handbook fulfills the proposed plan for this project and is available for use in the DSPS department at American River College.

APPENDICES

## APPENDIX A

An Algebra Handbook: Instructional Strategies for Assisting Students with Learning Disabilities

## TABLE OF CONTENTS

	Page
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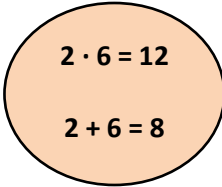
**SECTION 1: INTRODUCTION**

The handbook is designed to be a reference tool which will help Instructional Assistants and Learning Disability Specialists to better facilitate instructional strategies. It will enable instruction to be more effective in the learning process and promote academic achievement through improved strategies. The handbook consists of vital information on learning styles such as, auditory (learn by hearing), tactile/kinesthetic (learn by doing), and visual (learn by seeing or writing) [www.learning-styles-online.com/overview/](http://www.learning-styles-online.com/overview/). Additionally, the handbook provides information on asking students key questions to help in solving mathematical problems and understanding concepts. Detailed examples of strategies with visual illustrations are presented to help Instructional Assistants and Learning Disability Specialist demonstrate systematic techniques with the mathematical methods in problem solving during instructional strategy sessions with students with learning disabilities. A resource section provides references of books and online mathematical illustrations, learning styles, and instructional strategies which serve as supplementary support for the staff. Manipulative diagrams also are provided as tangible aids for staff while solving equations. The note section can be used by the reader to write down additional mathematical information as well as add notes about any personal teaching strategies that have been found to be effective.

## SECTION 2: MATHEMATICAL STRATEGIES WITH VISUAL ILLUSTRATIONS

Directions	Visualize	Visualize	Visualize	Conclusion								
Multiply two binomials $(x + 2)(x + 3)$	<b>Step 1:</b> Find the area of each rectangle. Top left: $x \cdot x = x^2$ Top right: $2 \cdot x = 2x$ Bottom left: $3 \cdot x = 3x$ Bottom right: $3 \cdot 2 = 6$	<b>Step 2:</b> Put a binomial on each side. $x + 2$ $x$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x \cdot x</math></td> <td><math>2 \cdot x</math></td> </tr> <tr> <td><math>3 \cdot x</math></td> <td><math>3 \cdot 2</math></td> </tr> </table> $+$ $3$	$x \cdot x$	$2 \cdot x$	$3 \cdot x$	$3 \cdot 2$	<b>Step 3:</b> Multiply. $x + 2$ $x$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>= x^2</math></td> <td><math>= 2x</math></td> </tr> <tr> <td><math>= 3x</math></td> <td><math>= 6</math></td> </tr> </table> $+$ $3$	$= x^2$	$= 2x$	$= 3x$	$= 6$	<b>Step 4:</b> Add the rectangles together. $x^2 + 2x + 3x + 6$ Combine like terms: $2x + 3x = 5x$ $x^2 + 2x + 3x + 6$ $x^2 + 5x + 6$ <b>Answer:</b> $(x + 2)(x + 3) = x^2 + 5x + 6$
$x \cdot x$	$2 \cdot x$											
$3 \cdot x$	$3 \cdot 2$											
$= x^2$	$= 2x$											
$= 3x$	$= 6$											

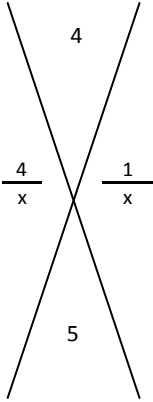


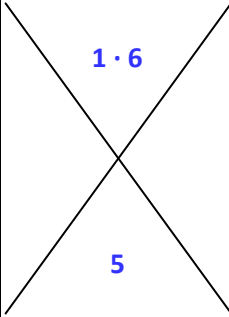
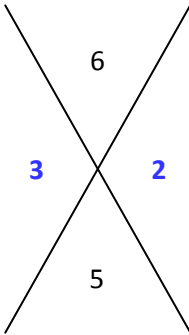
Directions	Visualize	Visualize	Visualize	Conclusion
Factor the polynomial $x^2 + 8x + 12$  Note: the first term must be $x^2$ .	<b>Step 1:</b> Find two numbers when <b>multiplied</b> together equal 12.  $1 \cdot 12 = 12$  $2 \cdot 6 = 12$ ← $3 \cdot 4 = 12$	<b>Step 2:</b> Find two numbers when <b>added</b> together equal 8.  $1 + 12 = 13$ $2 + 6 = 8$ ↔ $3 + 4 = 7$	<b>Step 3:</b> The numbers that satisfy both requirements are <b>2</b> and <b>6</b> .  	<b>Step 4:</b> Use the numbers 2 and 6 to write the factors of $x^2 + 8x + 12$ .  Factor: $x^2 + 8x + 12$  <b>Answer:</b> $(x + 2)(x + 6)$  Check:  Multiply $(x + 2)(x + 6)$ using the previous strategy (with rectangles) to arrive at $x^2 + 8x + 12$ .

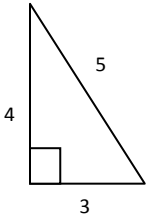
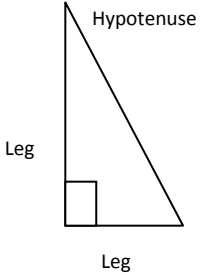
Directions	Visualize	Visualize	Conclusion
<b>Algebra Tools:</b>			
Addition Principle	→ $5 = 5$	→ $5 + 1 = 5 +$	This is the Addition Principle
	or	1	
Addition Principle using variables	→ $x = x$	→ or	This is the Addition Principle using the variable x
		$x + 1 = x$ + 1	
<b>Algebra Tools:</b>			
Subtraction Principle	→ $6 = 6$	→ $6 - 2 = 6 - 2$	This is the Subtraction Principle
	or	or	
Subtraction Principle using variables	→ $x = x$	→ $x - 2 = x - 2$	This is the Subtraction Principle using the variable x

Directions	Visualize	Visualize	Conclusion
<b>Algebra Tools:</b>  Multiplication Principle  Multiplication Principle using variables	$\rightarrow 7 = 7$ or $\rightarrow x = x$	$\rightarrow 7 \cdot 4 = 7 \cdot 4$ or $\rightarrow x \cdot 4 = x \cdot 4$	This is the Multiplication Principle  This is the Multiplication Principle using the variable x
<b>Algebra Tools:</b>  Division Principle  Division Principle using variables	$\rightarrow 10 = 10$ or $\rightarrow x = x$	$\rightarrow 10 \div 3 = 10 \div 3$ or $\rightarrow x \div 3 = x \div 3$	This is the Division Principle  This is the Division Principle using the variable x

Directions	Visualize	Visualize	Conclusion
Addition/ Subtraction Principle	If $3 = 3$	Then $3 + 4 = 3 + 4$	This is the Addition/Subtraction Principle
	If $9 = 9$	Then $9 - 8 = 9 - 8$	
	If $x = x$	Then $x + 1 = x + 1$	
Multiplicatio n/Division Principle	If $6 = 6$	Then $6 \cdot 2 = 6 \cdot 2$	This is the Multiplication/Division Principle
	If $12 = 12$	Then $(12) \div (3)$ $= (12) \div (3)$	
	If $10 = 10$	Then $(10) \div (2)$ $= (10) \div (2)$	
	If $x = x$	Then $(x) \div (8)$ $= (x) \div (8)$	

Directions	Visualize	Visualize	Visualize	Conclusion
<p>Solve the quadratic equation</p> $x^2 + 5x + 4 = 0$	<p><b>Step 1.</b> Find the factors.</p>  <p>The factors are:</p> $(x + 4)(x + 1)$	<p><b>Step 2.</b> Set each factor equal to zero and solve for x using the principles.</p> $\begin{array}{rcl} x + 4 & = & 0 \\ -4 & & -4 \\ \hline x & = & -4 \end{array}$ $\begin{array}{rcl} x + 1 & = & 0 \\ -1 & & -1 \\ \hline x & = & -1 \end{array}$	<p><b>Answer:</b> The solutions to the quadratic equation</p> $x = -4 \text{ and } x = -1$	<p><b>Step 4.</b> Check if <math>x = -4</math>.</p> $(-4)^2 + 5(-4) + 4 = 0$ $16 - 20 + 4 = 0$ $-4 + 4 = 0$ <p>True statement</p> <p><b>Step 5.</b> Check if <math>x = -1</math>.</p> $(-1)^2 + 5(-1) + 4 = 0$ $1 - 5 + 4 = 0$ $-4 + 4 = 0$ <p>True statement</p>

Directions	Visualize	Visualize	Visualize	Conclusion												
Solve the quadratic equation  $x^2 + 5x + 6 = 0$	<b>Step 1.</b> Identify the coefficients.  $1x^2 + 5x + 6 = 0$  First coefficient = 1 Second coefficient = 5 Third coefficient = 6  <b>Step 2.</b> Multiply the first and third coefficients. Place the result at the top of the X.	<b>Step 3.</b> Place the second coefficient at the bottom of the X.  	<b>Step 4.</b> Find two numbers whose product is 6 and whose sum is 5.    <b>Try 6 and 1:</b> <table border="1" data-bbox="883 989 1192 1079"> <thead> <tr> <th>Product</th> <th>Sum</th> <th>Match?</th> </tr> </thead> <tbody> <tr> <td><math>6 \cdot 1 = 6</math></td> <td><math>6 + 1 = 7</math></td> <td>No</td> </tr> </tbody> </table> <b>Try 3 and 2:</b> <table border="1" data-bbox="883 1199 1192 1289"> <thead> <tr> <th>Product</th> <th>Sum</th> <th>Match?</th> </tr> </thead> <tbody> <tr> <td><math>3 \cdot 2 = 6</math></td> <td><math>3 + 2 = 5</math></td> <td>Yes</td> </tr> </tbody> </table>	Product	Sum	Match?	$6 \cdot 1 = 6$	$6 + 1 = 7$	No	Product	Sum	Match?	$3 \cdot 2 = 6$	$3 + 2 = 5$	Yes	<b>Step 5.</b> Write the factors.  $(x + 3)(x + 2) = 0$  <b>Step 6.</b> Solve the each factor using the algebra principles.  $(x + 3) = 0$ and $(x + 2) = 0$  Solve $(x + 3) = 0$ $(x + 3) = 0$ $(x + 3) - 3 = 0 - 3$ $x + 3 - 3 = -3$ $x + 0 = -3$ $x = -3$  Solve $(x + 2) = 0$ $(x + 2) = 0$ $(x + 2) - 2 = 0 - 2$ $x + 2 - 2 = -2$ $x + 0 = -2$ $x = -2$  <b>Answer:</b> The solutions to the quadratic equation  $x = -3$ and $x = -2$
Product	Sum	Match?														
$6 \cdot 1 = 6$	$6 + 1 = 7$	No														
Product	Sum	Match?														
$3 \cdot 2 = 6$	$3 + 2 = 5$	Yes														

Directions	Visualize	Visualize	Conclusion
<p>Prove the triangle is a right triangle</p> 	<p>For all right triangles:</p> 	<p>Pythagorean Theorem:</p> $(\text{Leg})^2 + (\text{Leg})^2 = (\text{Hypotenuse})^2$	<p>Proof:</p> $(3)^2 + (4)^2 = (5)^2$ $9 + 16 = 25$ $25 = 25$ <p>True statement</p>

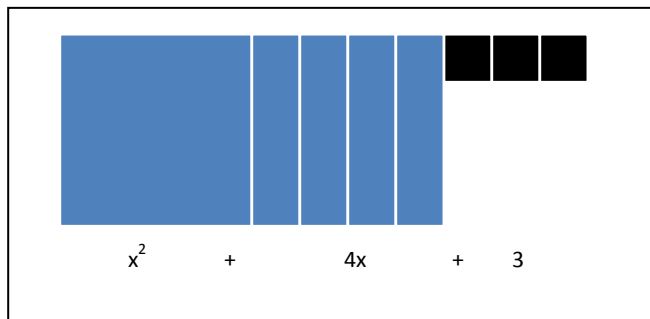
Directions	Visualize	Visualize	Visualize	Conclusion
<p><b>Word problems:</b> Two given facts and two unknowns</p> <p>Problem: “Audrey has three times the amount of money that Lori has. Together they have \$36. How much money does each person have?”</p>	<p><b>Step 1:</b> Identify the facts and the unknowns. Read the problem carefully.</p> <p><u>Fact 1.</u> Audrey has three times the amount of money that Lori has.</p> <p><u>Fact 2.</u> Together they have \$36.</p> <p><u>Unknown 1.</u> The amount of money Audrey has.</p> <p><u>Unknown 2:</u> The amount of money Lori has.</p>	<p><b>Step 2:</b> Choose a variable to represent one of the unknown quantities. Write an expression for the other unknown quantity using that variable and one of the facts.</p> <p>Hint: Chose the smaller quantity first. Lori has less money than Audrey, so use Lori’s information first.</p> <p><u>Use Unknown 2:</u> Lori has <b>h</b> dollars. Let Lori’s amount = <b>h</b>.</p> <p><u>Use Fact 1:</u> Audrey has three times the amount of money that Lori has. Audrey’s amount = <b>3h</b>.</p>	<p><b>Step 3:</b> Use the other fact to write an equation.</p> <p><u>Use Fact 2:</u> Together they have \$36.</p> <p>Lori’s amount + Audrey’s amount = 36</p> <p><b>h + 3h = 36</b></p>	<p><b>Step 4:</b> Solve the equation.</p> <p><b>h + 3h = 36</b></p> <p><b>4h = 36</b></p> <p><b>h = 36 ÷ 4</b></p> <p><b>h = 9</b></p> <p><b>Answer:</b> Lori has <b>h = 9</b> dollars, and Audrey has <b>3h = 3(9) = 27</b> dollars.</p> <p>Check the answer: <b>9 + 27 = 36</b></p> <p><b>True Statement</b></p>



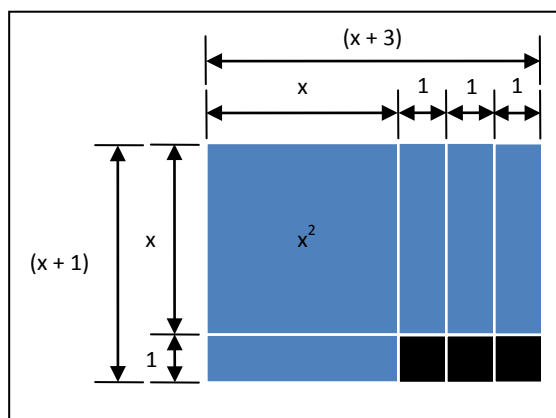
Directions	Visualize	Visualize	Visualize	Conclusion
<p><b>Word problems:</b> Two given facts and two unknowns</p> <p><u>Problem:</u> "John has \$20 more than Andrew. Together they have \$100. How much money does each person have?"</p>	<p><b>Step 1:</b> Identify the facts and the unknowns. Read the problem carefully.</p> <p><u>Fact 1.</u> John has \$20 more than Andrew.</p> <p><u>Fact 2.</u> Together they have \$100.</p> <p><u>Unknown 1.</u> The amount of money John has.</p> <p><u>Unknown 2:</u> The amount of money Andrew has.</p>	<p><b>Step 2:</b> Choose a variable to represent one of the unknown quantities. Write an expression for the other unknown quantity using that variable and one of the facts.</p> <p><u>Hint:</u> Chose the smaller quality first. Andrew has less money than John, so use John's information first.</p> <p><u>Use Unknown 2:</u> Andrew has <b>p</b> dollars. Let Andrew's amount = <b>p</b>.</p> <p><u>Use Fact 1:</u> John has \$20 more than Andrew. John's amount = <b>p + 20</b>.</p>	<p><b>Step 3:</b> Use the other fact to write an equation.</p> <p><u>Use Fact 2:</u> Together they have \$100.</p> <p>Andrew's amount + John's amount = 100</p> $p + (p + 20) = 100$ $p + p + 20 = 100$ $2p + 20 = 100$	<p><b>Step 4:</b> Solve the equation.</p> $2p + 20 = 100$ $2p = 100 - 20$ $2p = 80$ $p = 80 \div 2$ $p = 40$ <p><b>Answer:</b> Andrew has <b>p = 40</b> dollars, and John has <b>p + 20 = 40 + 20 = 60</b> dollars.</p> <p>Check the answer:</p> $2(40) + 20 = 100$ $80 + 20 = 100$ <p>True Statement</p>

Directions	Visualize	Visualize	Visualize	Conclusion																				
<p><b>Word problems:</b> Uniform Motion (Rate-Time-Distance)</p> <p>Problem:</p> <p>“Two cars start 60 miles apart at 12:00 PM and drive towards each other at a constant speed. They meet up one half-hour (0.5 hours) later at 12:30 PM. The first car (Car A) travels at a speed that is twice that of the other car (Car B). Find the speed of each car.”</p>	<p>Rate-Time-Distance equation:</p> <p><b>Distance = Rate x Time</b></p> <p>Distance: <b>D</b> miles</p> <p>Rate: <b>r</b> miles per hour</p> <p>Time: <b>t</b> hours</p> <p><b>D = rt</b></p> <p><b>Step 1:</b> Sketch the problem.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Car A</td> <td>Car B</td> </tr> <tr> <td>rate 2r →</td> <td>rate r ←</td> </tr> <tr> <td>Dist. A</td> <td>Dist. B</td> </tr> <tr> <td colspan="2" style="text-align: center;">60 miles total distance</td> </tr> </table> <p>Dist. A + Dist. B = 60 miles</p>	Car A	Car B	rate 2r →	rate r ←	Dist. A	Dist. B	60 miles total distance		<p><b>Step 2:</b> Make a chart to organize the facts.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Car</td> <td>Rate</td> <td>Time</td> <td>Dist.</td> </tr> <tr> <td>A</td> <td>2r</td> <td>0.5</td> <td>(2r)(0.5)</td> </tr> <tr> <td>B</td> <td>r</td> <td>0.5</td> <td>(r)(0.5)</td> </tr> </table> <p>Distance A = Rate x Time</p> <p>Distance A = (2r)(0.5)</p> <p>Distance B = Rate x Time</p> <p>Distance B = (r)(0.5)</p>	Car	Rate	Time	Dist.	A	2r	0.5	(2r)(0.5)	B	r	0.5	(r)(0.5)	<p><b>Step 3:</b> Write an equation.</p> <p>Distance A + Distance B = 60 miles</p> $(2r)(0.5) + (r)(0.5) = 60$ $(0.5)(2r + r) = 60$ $(0.5)(3r) = 60$	<p><b>Step 4:</b> Solve for <b>r</b>.</p> $(0.5)(3r) = 60$ $3r = 60 \div 0.5$ $r = 120 \div 3$ $r = 40$ <p><b>Answer:</b></p> <p>Car A travels at <math>2r = 2(40) = 80</math> miles per hour.</p> <p>Car B travels at <math>r = 40</math> miles per hour.</p>
Car A	Car B																							
rate 2r →	rate r ←																							
Dist. A	Dist. B																							
60 miles total distance																								
Car	Rate	Time	Dist.																					
A	2r	0.5	(2r)(0.5)																					
B	r	0.5	(r)(0.5)																					

### Graphical representation of polynomials



**Figure 1:** Graphical representation of  $x^2 + 4x + 3$ .



**Figure 2:** Graphical representation of  $(x + 1)(x + 3)$ .

**Problem:** Show that the polynomial  $x^2 + 4x + 3$  is the product of  $(x + 1)$  and  $(x + 3)$ .

**Solution:**

**Step 1:** Start with  $x^2 + 4x + 3$ .

**Step 2:** Draw one  $x$ -by- $x$  square, four  $x$ -by-1 rectangles, and three 1-by-1 squares. This is a total of eight shapes. See Figure 1.

**Step 3:** Arrange these eight shapes into one complete rectangle. See Figure 2.

**Step 4:** Mark the sides of the complete rectangle as  $(x + 1)$  and  $(x + 3)$ . This shows that the area of the complete rectangle is  $(x + 1)$  by  $(x + 3)$ .

### Multiply two binomials

Multiply two binomials  $(x + 2)(x + 3)$ .

#### Step 1

Top left rectangle  $\rightarrow x \cdot x = x^2$

Top right rectangle  $\rightarrow 2 \cdot x = 2x$

Bottom left rectangle  $\rightarrow 3 \cdot x = 3x$

Bottom right rectangle  $\rightarrow 3 \cdot 2 = 6$

#### Step 2

	$x + 2$	
$x$	$x \cdot x$	$2 \cdot x$
+		
$3$	$3 \cdot x$	$3 \cdot 2$

#### Step 4

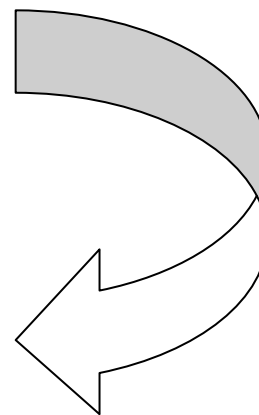
$$x^2 + 2x + 3x + 6$$

**Combine**  $2x + 3x = 5x$

**Answer:**  $(x + 2)(x + 3) = x^2 + 5x + 6$

#### Step 3

	$x + 2$	
$x$	$=x^2$	$=2x$
+		
$3$	$=3x$	$=6$



**Solution:**      **Step 1:** Find the area of each rectangle.

**Step 2:** Put a binomial on each side.

**Step 3:** Multiply.

**Step 4:** Add each rectangle together.

### SECTION 3: MATHEMATICAL RESOURCES

Internet:

Virtual Math Lab: Intermediate Algebra  
The University of West Texas A&M

[http://www.wtamu.edu/academic/anns/mps/math/mathlab/int\\_algebra/index.htm](http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/index.htm)

The Math League

<http://www.mathleague.com/help/algebra/algebra.htm>

Coolmath Algebra (1997-2010)

<http://www.coolmath.com/algebra/>

Understanding Algebra: An Online Algebra text

<http://www.jamesbrennan.org/algebra/>

LDonline

[www.ldonline.org/](http://www.ldonline.org/)

Mathematical Symbols: EnchantedLearning. com

[www.enchantedlearning.com/math/symbols/](http://www.enchantedlearning.com/math/symbols/)

Books:

Becoming a Master Student (11<sup>th</sup> ed.)

D. Ellis (2006). Boston, MA: Houghton Mifflin.

Classroom Discussion: Using Math Talk To Help Students Learn

S. Chapin, C. O'Connor, & N. Anderson (2003). Sausalito, CA: Math Solutions

You are Smarter than you Think (2<sup>nd</sup> ed.)

R. Mollan-Masters (1997). Ashland, OR: Harper Collins

Master Math Solving Word Problems

Brita Immergut (2010). Boston, MA: Course Technology.

College Algebra: Demystified

Rhonda Huettenmueller (2004). New York, NY: McGraw-Hill.

The Humougeous Book of Algebra Problems

W. Michael Kelley (2008). Indianapolis, IN: ALPHA Books.

**SECTION 4: MANIPULATIVE DIAGRAMS**

Diagram of 0.5" Squares Grid

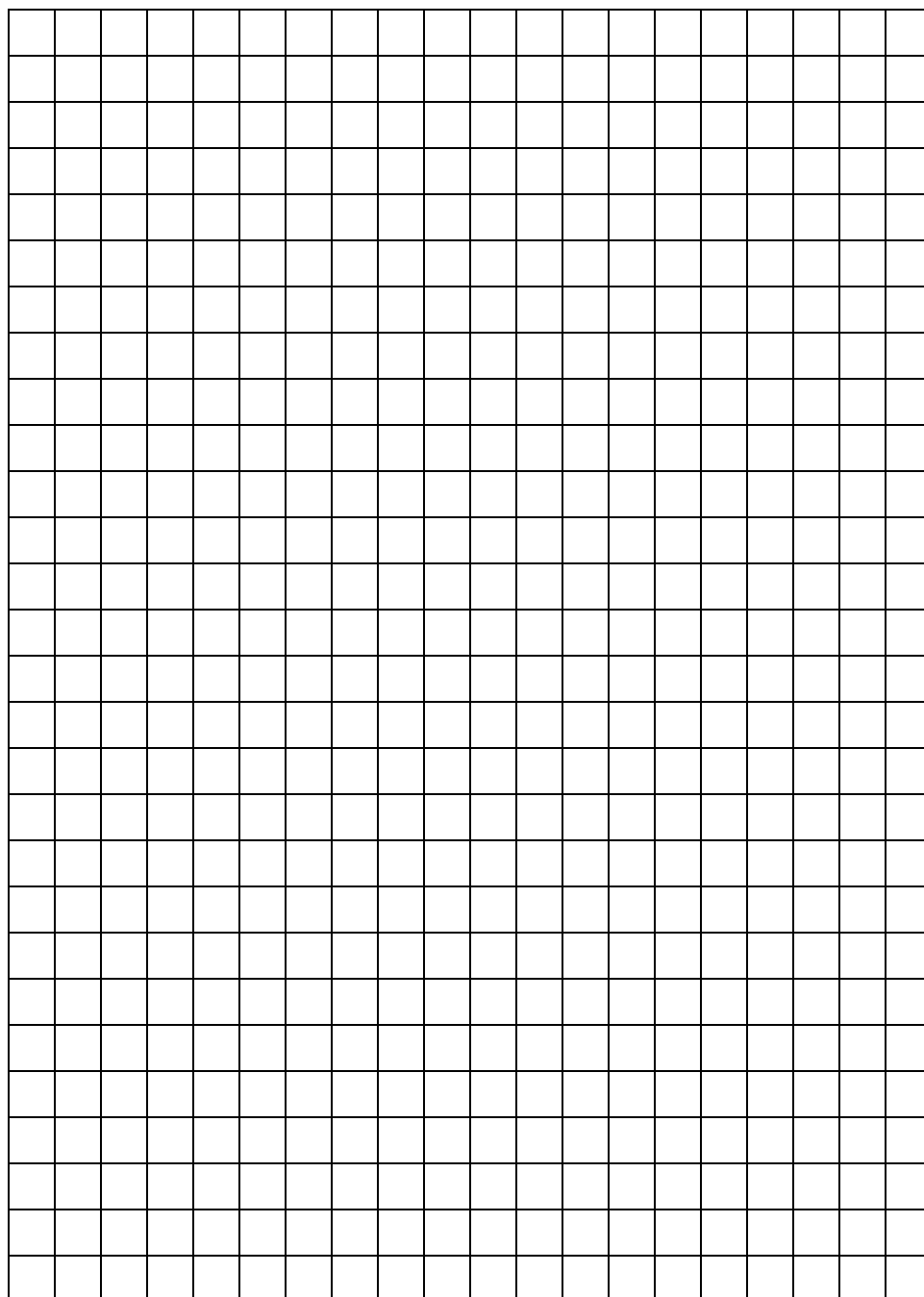
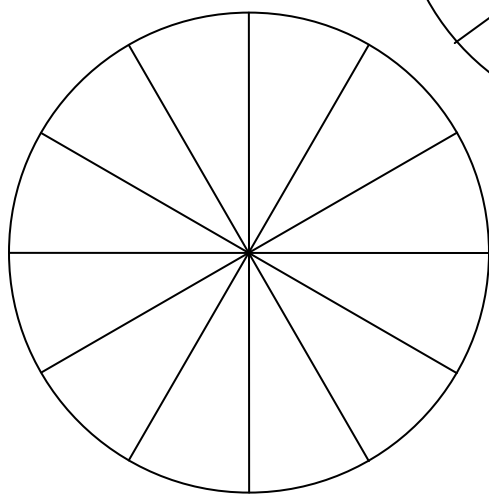
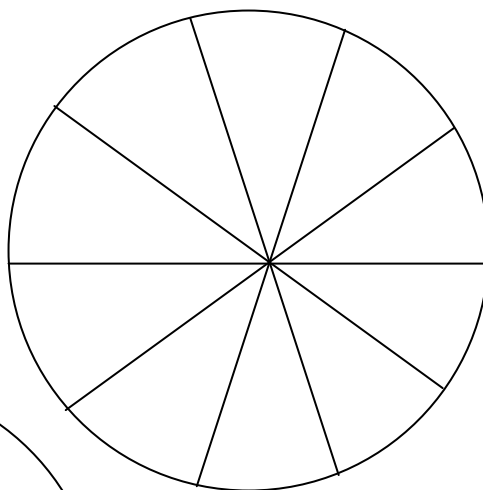
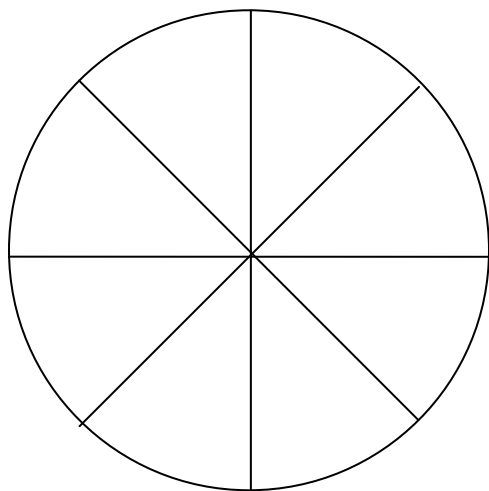
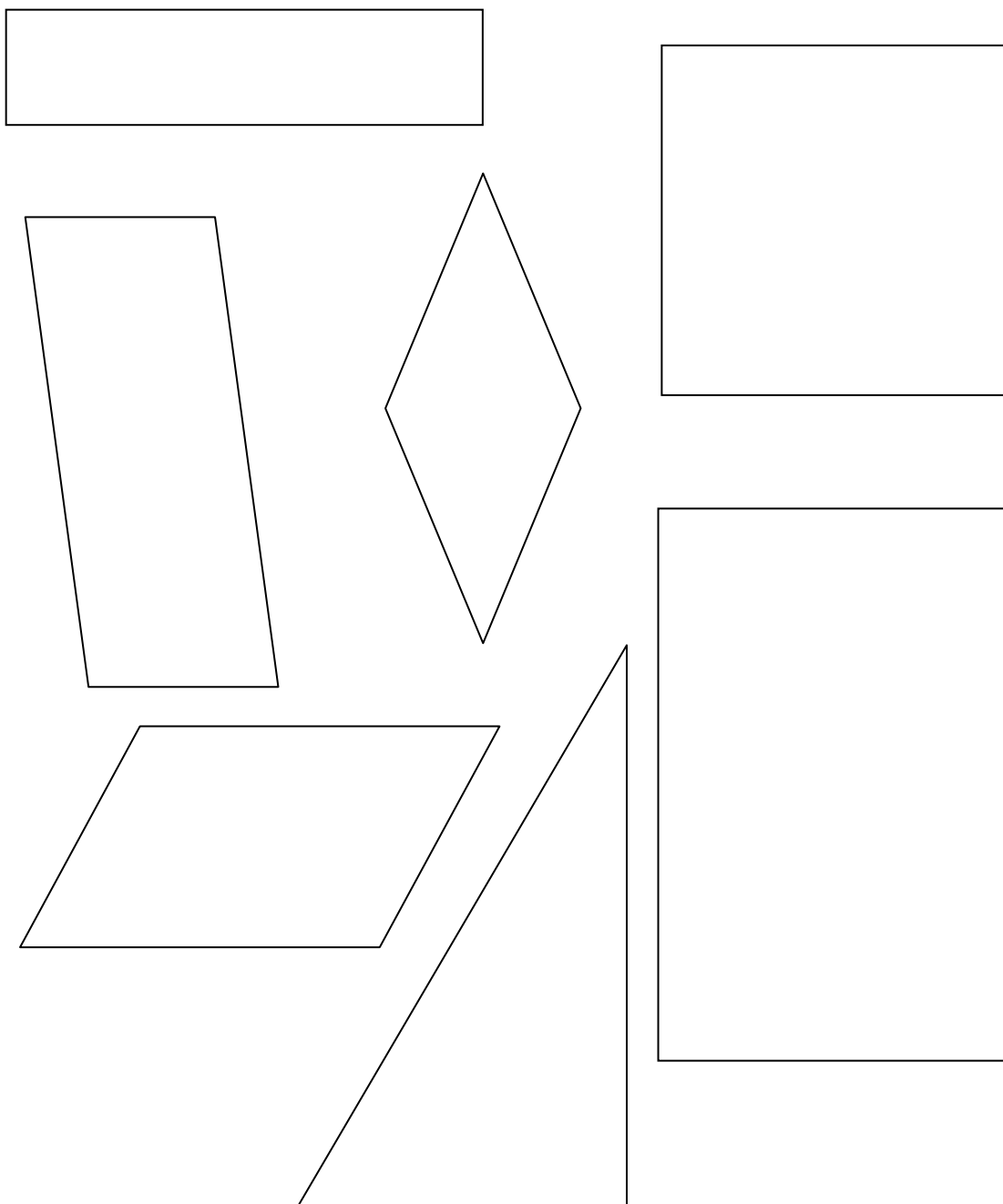


Diagram of Circular Fraction Pieces



## Diagram of Assorted Shapes





**SECTION 5: NOTES**

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